Active contours for image segmentation

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Detect and isolate objects in image u_0

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Detect and isolate objects in image u_0

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Evolve contour C s.t. stops on boundary of object

Active contours *with* edges

Active contours with edges

Edge-detector-based snake/active contour model[1]:

$$C^* = \arg\inf_{C} \left\{ J(C) \right\}$$

[1] Kass, Witkin, and Terzopoulos 1988.

Active contours with edges

Edge-detector-based snake/active contour model[1]:

$$C^* = \arg\inf_C \left\{ \underbrace{\alpha \int_0^1 \left\|C'(s)\right\|_2^2 \, \mathrm{d}s + \beta \int_0^1 \left\|C''(s)\right\|_2^2 \, \mathrm{d}s}_{\text{smoothness of contour}} \right\}$$

where $\alpha, \beta, \lambda \in \mathbb{R}_{\geq 0}$ and $C(s) : [0,1] \to \mathbb{R}^2$

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Edge-detector-based snake/active contour model[1]:

$$C^* = \arg\inf_C \left\{ \underbrace{\alpha \int_0^1 \|C'(s)\|_2^2 \, ds + \beta \int_0^1 \|C''(s)\|_2^2 \, ds}_{\text{internal energy}} - \lambda \underbrace{\int_0^1 \|\nabla u_0(C(s))\|_2^2 \, ds}_{\text{external energy}} \right\}$$

where $\alpha, \beta, \lambda \in \mathbb{R}_{\geq 0}$ and $C(s) : [0, 1] \to \mathbb{R}^2$

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Active contours with edges

How to solve:

$$C^* = \arg\inf_C J(C)$$

where C is a function?

Active contours with edges

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Calculus of variations

Active contours with edges

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$$C^* = \arg\inf_C J(C)$$

where C is a function?

Calculus of variations

$$\left.\frac{d}{d\epsilon}\right|_{\epsilon=0} J(C^* + \epsilon\eta) = 0$$

Active contours with edges

How to solve:

$$C^* = \arg\inf_C J(C)$$

where C is a function?

Calculus of variations $\rightarrow$ Euler-Lagrange equation

$$\frac{\partial \phi}{\partial t} = \|\nabla \phi\| \, F, \quad \phi(0, x, y) = \phi_0(x, y), \quad C = \{(x, y) \mid \phi(\cdot, x, y) = 0\}$$

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Want stationary solution of the differential equation

Active contours with edges

Key assumption

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Maxima of $|\nabla u_0|$ are edges

Active contours with edges

Key assumption

Maxima of $|\nabla u_0|$ are edges

Implementation

Evolve C based on internal forces and edge map $g(u_0)$

Active contours with edges

$u_0(x, y)$

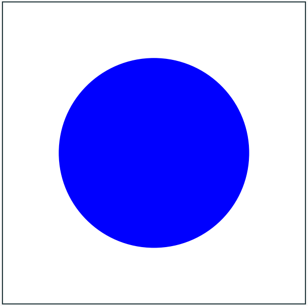
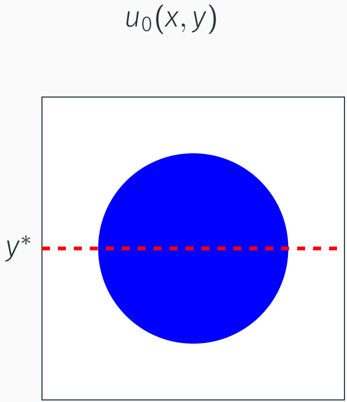


Figure 1: Example image

Active contours with edges



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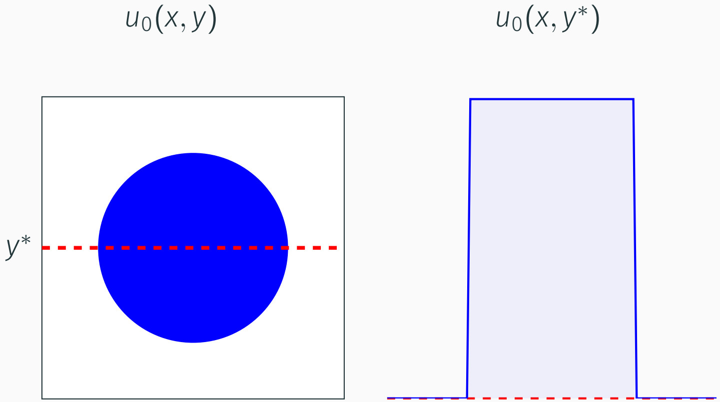

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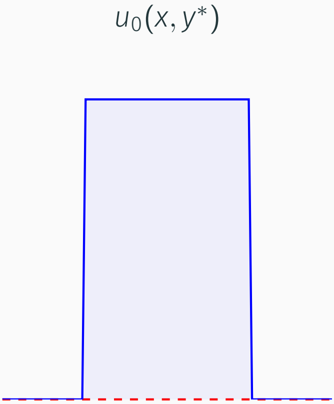
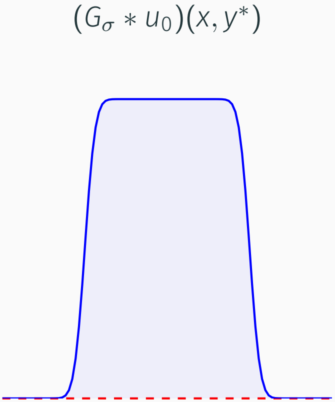


Figure 2: Gradient-based segmentation with active contours

Active contours with edges



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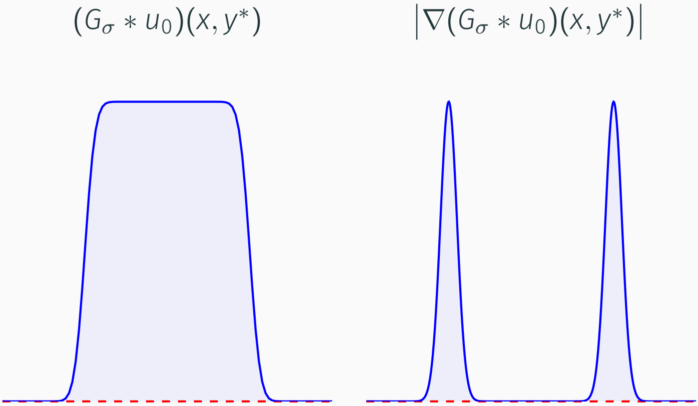
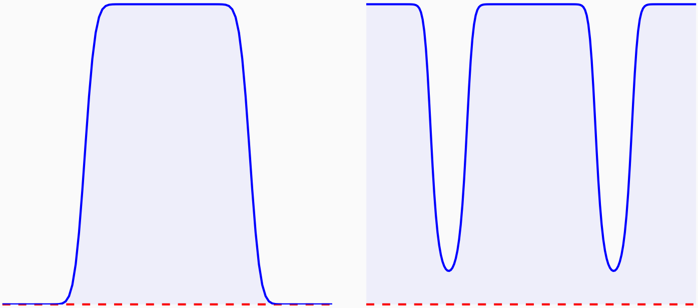


Figure 2: Gradient-based segmentation with active contours

Active contours with edges

$(G_\sigma * u_0)(x, y^*)$ $\qquad$ $\frac{1}{1+\left|\nabla(G_\sigma * u_0)(x,y^*)\right|}$



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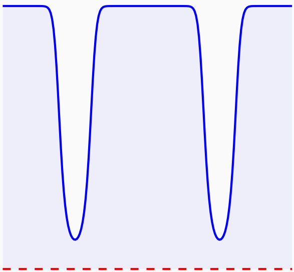
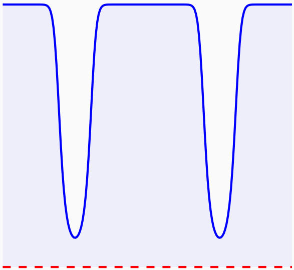


[2] Caselles et al. 1993.

Active contours with edges

$g(u_0)$



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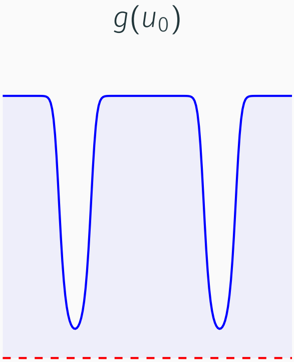

Geometric active contour[2]:

$$\frac{\partial \phi}{\partial t} = g(u_0)\,(\kappa + \nu)$$

where

$$\kappa = \nabla \cdot \left(\frac{\nabla \phi}{\|\nabla \phi\|} \right), \ \nu \in \mathbb{R}_{\geq 0}$$

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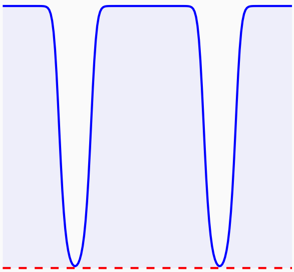

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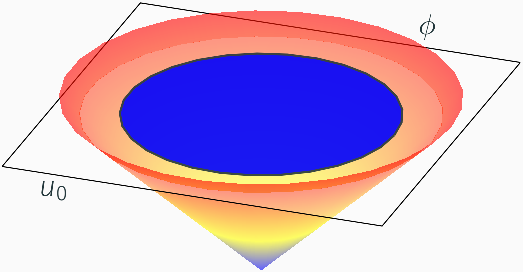

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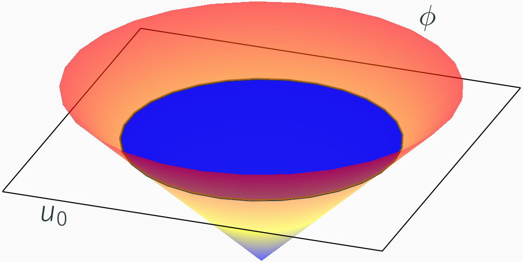


Active contours *without* edges

$(G_\sigma * u_0)(x, y^*)$

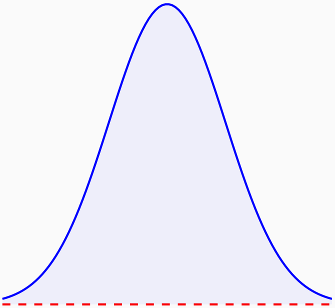
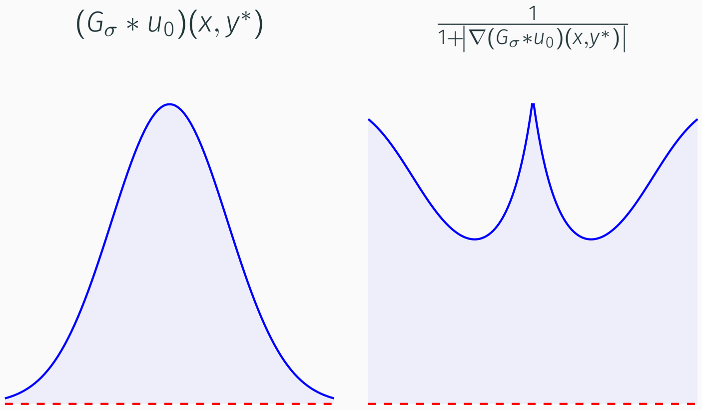


Figure 3: Gradient-based segmentation with active contours

Motivation



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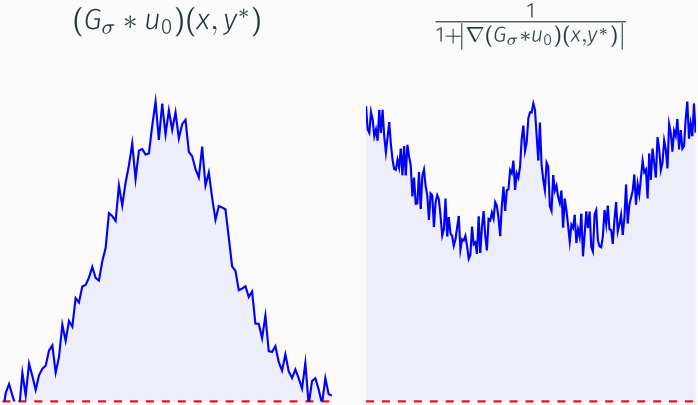

Figure 3: Gradient-based segmentation with active contours

Approach

Chan and Vese[3] propose:

Core concept

Evolve C s.t. two constants—associated with $\mathrm{int}(C)$ and $\mathrm{ext}(C)$—best approximate u_0

[3] Chan and Vese 2001.

Energy functional $F(c_1, c_2, C)$:

$$
\begin{aligned}
F(c_1, c_2, C) = & \mu \cdot \mathrm{Length}(C) \\
& \nu \cdot \mathrm{Area}(\mathrm{int}(C)) \\
& + \lambda_1 \int_{\mathrm{int}(C)} \left| u_0(x, y) - c_1 \right|^2 \mathrm{d}x \, \mathrm{d}y \\
& + \lambda_2 \int_{\mathrm{ext}(C)} \left| u_0(x, y) - c_2 \right|^2 \mathrm{d}x \, \mathrm{d}y
\end{aligned}
$$

where $\mu, \nu \geq 0$ and $\lambda_1, \lambda_2 > 0$

Goal

Find C^* such that:

$$c_1^*, c_2^*, C^* = \arg\inf_{c_1, c_2, C} F(c_1, c_2, C)$$

[4] Mumford and Shah 1989.

Active contours without edges

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Find C^* such that:

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Remark

Mumford-Shah[4] provide proof of minimizer existence

[4] Mumford and Shah 1989.

Active contours without edges

Solve the calculus of variations problem

Solve the calculus of variations problem

Euler-Lagrange equation for ϕ:

$$\frac{\partial \phi}{\partial t} = \delta_\varepsilon(\phi)\left[\mu\,\kappa - \nu - \lambda_1(u_0 - c_1)^2 + \lambda_2(u_0 - c_2)^2\right] = 0$$

and

$$\phi(0, x, y) = \phi_0(x, y) \qquad \frac{\delta_\varepsilon(\phi)}{\|\lambda\phi\|}\frac{\partial \phi}{\partial \vec{n}} = 0 \text{ on } \partial\Omega.$$

Algorithm Active contour without edges

procedure CHAN-VESE(u_0, ϕ_0)

$\phi^0 \leftarrow \phi_0$

Algorithm Active contour without edges

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$\phi^0 \leftarrow \phi_0$

$n \leftarrow 1$

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while solution not stationary **do**

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$c_1 \leftarrow \mathrm{average}(\mathrm{int}(C))$

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$\phi^{n+1} \leftarrow$ solve Euler-Lagrange equation for ϕ

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    Reinitialize ϕ for signed distance function (optional)

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    Reinitialize ϕ for signed distance function (optional)

    $n \leftarrow n + 1$

  return $\mathrm{sign}(\phi^{\text{final}})$ on u_0

Demonstration

Demo